

# 1 REVIEW EXERCISES

See [www.CalcChat.com](http://www.CalcChat.com) for worked-out solutions to odd-numbered exercises.

In Exercises 1 and 2, determine whether the problem can be solved using precalculus or if calculus is required. If the problem can be solved using precalculus, solve it. If the problem seems to require calculus, explain your reasoning. Use a graphical or numerical approach to estimate the solution.

- Find the distance between the points (1, 1) and (3, 9) along the curve  $y = x^2$ .
- Find the distance between the points (1, 1) and (3, 9) along the line  $y = 4x - 3$ .

In Exercises 3 and 4, complete the table and use the result to estimate the limit. Use a graphing utility to graph the function to confirm your result.

$x$	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

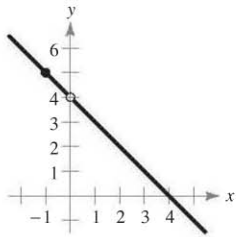
- $\lim_{x \rightarrow 0} \frac{[4/(x+2)] - 2}{x}$
- $\lim_{x \rightarrow 0} \frac{4(\sqrt{x+2} - \sqrt{2})}{x}$

In Exercises 5–8, find the limit  $L$ . Then use the  $\epsilon$ - $\delta$  definition to prove that the limit is  $L$ .

- $\lim_{x \rightarrow 1} (x + 4)$
- $\lim_{x \rightarrow 9} \sqrt{x}$
- $\lim_{x \rightarrow 2} (1 - x^2)$
- $\lim_{x \rightarrow 5} 9$

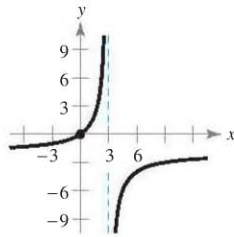
In Exercises 9 and 10, use the graph to determine each limit.

9.  $h(x) = \frac{4x - x^2}{x}$



- (a)  $\lim_{x \rightarrow 0} h(x)$       (b)  $\lim_{x \rightarrow -1} h(x)$

10.  $g(x) = \frac{-2x}{x - 3}$



- (a)  $\lim_{x \rightarrow 3} g(x)$       (b)  $\lim_{x \rightarrow 0} g(x)$

In Exercises 11–26, find the limit (if it exists).

- $\lim_{x \rightarrow 6} (x - 2)^2$
- $\lim_{x \rightarrow 7} (10 - x)^4$
- $\lim_{t \rightarrow 4} \sqrt{t + 2}$
- $\lim_{y \rightarrow 4} 3|y - 1|$
- $\lim_{t \rightarrow -2} \frac{t + 2}{t^2 - 4}$
- $\lim_{t \rightarrow 3} \frac{t^2 - 9}{t - 3}$
- $\lim_{x \rightarrow 4} \frac{\sqrt{x - 3} - 1}{x - 4}$
- $\lim_{x \rightarrow 0} \frac{\sqrt{4 + x} - 2}{x}$

- $\lim_{x \rightarrow 0} \frac{[1/(x+1)] - 1}{x}$
- $\lim_{s \rightarrow 0} \frac{(1/\sqrt{1+s}) - 1}{s}$
- $\lim_{x \rightarrow -5} \frac{x^3 + 125}{x + 5}$
- $\lim_{x \rightarrow -2} \frac{x^2 - 4}{x^3 + 8}$
- $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin x}$
- $\lim_{x \rightarrow \pi/4} \frac{4x}{\tan x}$
- $\lim_{\Delta x \rightarrow 0} \frac{\sin[(\pi/6) + \Delta x] - (1/2)}{\Delta x}$   
[Hint:  $\sin(\theta + \phi) = \sin \theta \cos \phi + \cos \theta \sin \phi$ ]
- $\lim_{\Delta x \rightarrow 0} \frac{\cos(\pi + \Delta x) + 1}{\Delta x}$   
[Hint:  $\cos(\theta + \phi) = \cos \theta \cos \phi - \sin \theta \sin \phi$ ]

In Exercises 27–30, evaluate the limit given  $\lim_{x \rightarrow c} f(x) = -\frac{3}{4}$  and  $\lim_{x \rightarrow c} g(x) = \frac{2}{3}$ .

- $\lim_{x \rightarrow c} [f(x)g(x)]$
- $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$
- $\lim_{x \rightarrow c} [f(x) + 2g(x)]$
- $\lim_{x \rightarrow c} [f(x)]^2$

*Numerical, Graphical, and Analytic Analysis* In Exercises 31 and 32, consider

$\lim_{x \rightarrow 1^+} f(x)$ .

- Complete the table to estimate the limit.
- Use a graphing utility to graph the function and use the graph to estimate the limit.
- Rationalize the numerator to find the exact value of the limit analytically.

$x$	1.1	1.01	1.001	1.0001
$f(x)$				

- $f(x) = \frac{\sqrt{2x+1} - \sqrt{3}}{x-1}$
- $f(x) = \frac{1 - \sqrt[3]{x}}{x-1}$

[Hint:  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ ]

*Free-Falling Object* In Exercises 33 and 34, use the position function  $s(t) = -4.9t^2 + 250$ , which gives the height (in meters) of an object that has fallen from a height of 250 meters. The velocity at time  $t = a$  seconds is given by

$\lim_{t \rightarrow a} \frac{s(a) - s(t)}{a - t}$ .

- Find the velocity of the object when  $t = 4$ .
- At what velocity will the object impact the ground?

In Exercises 35–40, find the limit (if it exists). If the limit does not exist, explain why.

35.  $\lim_{x \rightarrow 3^-} \frac{|x - 3|}{x - 3}$

36.  $\lim_{x \rightarrow 4} \lfloor x - 1 \rfloor$

37.  $\lim_{x \rightarrow 2} f(x)$ , where  $f(x) = \begin{cases} (x - 2)^2, & x \leq 2 \\ 2 - x, & x > 2 \end{cases}$

38.  $\lim_{x \rightarrow 1^+} g(x)$ , where  $g(x) = \begin{cases} \sqrt{1 - x}, & x \leq 1 \\ x + 1, & x > 1 \end{cases}$

39.  $\lim_{t \rightarrow 1} h(t)$ , where  $h(t) = \begin{cases} t^3 + 1, & t < 1 \\ \frac{1}{2}(t + 1), & t \geq 1 \end{cases}$

40.  $\lim_{s \rightarrow -2} f(s)$ , where  $f(s) = \begin{cases} -s^2 - 4s - 2, & s \leq -2 \\ s^2 + 4s + 6, & s > -2 \end{cases}$

In Exercises 41–52, determine the intervals on which the function is continuous.

41.  $f(x) = -3x^2 + 7$

42.  $f(x) = x^2 - \frac{2}{x}$

43.  $f(x) = \lfloor x + 3 \rfloor$

44.  $f(x) = \frac{3x^2 - x - 2}{x - 1}$

45.  $f(x) = \begin{cases} \frac{3x^2 - x - 2}{x - 1}, & x \neq 1 \\ 0, & x = 1 \end{cases}$

46.  $f(x) = \begin{cases} 5 - x, & x \leq 2 \\ 2x - 3, & x > 2 \end{cases}$

47.  $f(x) = \frac{1}{(x - 2)^2}$

48.  $f(x) = \sqrt{\frac{x + 1}{x}}$

49.  $f(x) = \frac{3}{x + 1}$

50.  $f(x) = \frac{x + 1}{2x + 2}$

51.  $f(x) = \csc \frac{\pi x}{2}$

52.  $f(x) = \tan 2x$

53. Determine the value of  $c$  such that the function is continuous on the entire real line.

$$f(x) = \begin{cases} x + 3, & x \leq 2 \\ cx + 6, & x > 2 \end{cases}$$

54. Determine the values of  $b$  and  $c$  such that the function is continuous on the entire real line.

$$f(x) = \begin{cases} x + 1, & 1 < x < 3 \\ x^2 + bx + c, & |x - 2| \geq 1 \end{cases}$$

55. Use the Intermediate Value Theorem to show that  $f(x) = 2x^3 - 3$  has a zero in the interval  $[1, 2]$ .



**56. Delivery Charges** The cost of sending an overnight package from New York to Atlanta is \$12.80 for the first pound and \$2.50 for each additional pound or fraction thereof. Use the greatest integer function to create a model for the cost  $C$  of overnight delivery of a package weighing  $x$  pounds. Use a graphing utility to graph the function and discuss its continuity.

57. Let  $f(x) = \frac{x^2 - 4}{|x - 2|}$ . Find each limit (if possible).

(a)  $\lim_{x \rightarrow 2^-} f(x)$

(b)  $\lim_{x \rightarrow 2^+} f(x)$

(c)  $\lim_{x \rightarrow 2} f(x)$

58. Let  $f(x) = \sqrt{x(x - 1)}$ .

(a) Find the domain of  $f$ .

(b) Find  $\lim_{x \rightarrow 0^-} f(x)$ .

(c) Find  $\lim_{x \rightarrow 1^+} f(x)$ .

In Exercises 59–62, find the vertical asymptotes (if any) of the graph of the function.

59.  $g(x) = 1 + \frac{2}{x}$

60.  $h(x) = \frac{4x}{4 - x^2}$

61.  $f(x) = \frac{8}{(x - 10)^2}$

62.  $f(x) = \csc \pi x$

In Exercises 63–74, find the one-sided limit (if it exists).

63.  $\lim_{x \rightarrow -2} \frac{2x^2 + x + 1}{x + 2}$

64.  $\lim_{x \rightarrow (1/2)^+} \frac{x}{2x - 1}$

65.  $\lim_{x \rightarrow -1^+} \frac{x + 1}{x^3 + 1}$

66.  $\lim_{x \rightarrow -1^-} \frac{x + 1}{x^4 - 1}$

67.  $\lim_{x \rightarrow 1} \frac{x^2 + 2x + 1}{x - 1}$

68.  $\lim_{x \rightarrow -1^+} \frac{x^2 - 2x + 1}{x + 1}$

69.  $\lim_{x \rightarrow 0^+} \left( x - \frac{1}{x^3} \right)$

70.  $\lim_{x \rightarrow 2^-} \frac{1}{\sqrt[3]{x^2 - 4}}$

71.  $\lim_{x \rightarrow 0^+} \frac{\sin 4x}{5x}$

72.  $\lim_{x \rightarrow 0^+} \frac{\sec x}{x}$

73.  $\lim_{x \rightarrow 0^+} \frac{\csc 2x}{x}$

74.  $\lim_{x \rightarrow 0^-} \frac{\cos^2 x}{x}$

**75. Environment** A utility company burns coal to generate electricity. The cost  $C$  in dollars of removing  $p\%$  of the air pollutants in the stack emissions is

$$C = \frac{80,000p}{100 - p}, \quad 0 \leq p < 100.$$

Find the costs of removing (a) 15%, (b) 50%, and (c) 90% of the pollutants. (d) Find the limit of  $C$  as  $p \rightarrow 100^-$ .

76. The function  $f$  is defined as shown.

$$f(x) = \frac{\tan 2x}{x}, \quad x \neq 0$$

(a) Find  $\lim_{x \rightarrow 0} \frac{\tan 2x}{x}$  (if it exists).

(b) Can the function  $f$  be defined at  $x = 0$  such that it is continuous at  $x = 0$ ?