7.3 Volume: The Shell Method

- **Find the volume of a solid of revolution using the shell method.**
- **Compare the uses of the disk method and the shell method.**

The Shell Method

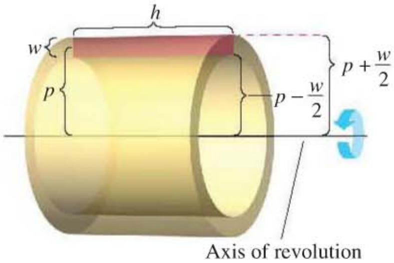


Figure 7.27

In this section, you will study an alternative method for finding the volume of a solid of revolution. This method is called the **shell method** because it uses cylindrical shells. A comparison of the advantages of the disk and shell methods is given later in this section.

To begin, consider a representative rectangle as shown in Figure 7.27, where w is the width of the rectangle, h is the height of the rectangle, and p is the distance between the axis of revolution and the *center* of the rectangle. When this rectangle is revolved about its axis of revolution, it forms a cylindrical shell (or tube) of thickness w. To find the volume of this shell, consider two cylinders. The radius of the larger cylinder corresponds to the outer radius of the shell, and the radius of the smaller cylinder corresponds to the inner radius of the shell. Because p is the average radius of the shell, you know the outer radius is $p + (w/2)$ and the inner radius is $p - (w/2)$.

$$p + \frac{w}{2} \qquad \text{Outer radius}$$

$$p - \frac{w}{2} \qquad \text{Inner radius}$$

So, the volume of the shell is

$$\begin{aligned}\text{Volume of shell} &= (\text{volume of cylinder}) - (\text{volume of hole})\\ &= \pi\left(p + \frac{w}{2}\right)^2 h - \pi\left(p - \frac{w}{2}\right)^2 h\\ &= 2\pi phw\\ &= 2\pi(\text{average radius})(\text{height})(\text{thickness}).\end{aligned}$$

You can use this formula to find the volume of a solid of revolution. Assume that the plane region in Figure 7.28 is revolved about a line to form the indicated solid. If you consider a horizontal rectangle of width Δy, then, as the plane region is revolved about a line parallel to the x-axis, the rectangle generates a representative shell whose volume is

$$\Delta V = 2\pi[p(y)h(y)]\,\Delta y.$$

You can approximate the volume of the solid by n such shells of thickness Δy, height $h(y_i)$, and average radius $p(y_i)$.

$$\text{Volume of solid} \approx \sum_{i=1}^{n} 2\pi[p(y_i)h(y_i)]\,\Delta y = 2\pi\sum_{i=1}^{n}[p(y_i)h(y_i)]\,\Delta y$$

This approximation appears to become better and better as $\|\Delta\| \to 0$ $(n \to \infty)$. So, the volume of the solid is

$$\begin{aligned}\text{Volume of solid} &= \lim_{\|\Delta\|\to 0} 2\pi\sum_{i=1}^{n}[p(y_i)h(y_i)]\,\Delta y\\ &= 2\pi\int_c^d [p(y)h(y)]\,dy.\end{aligned}$$

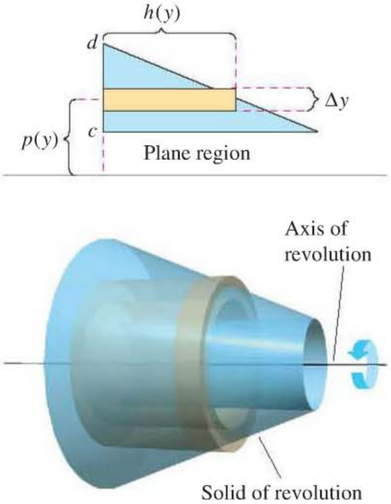
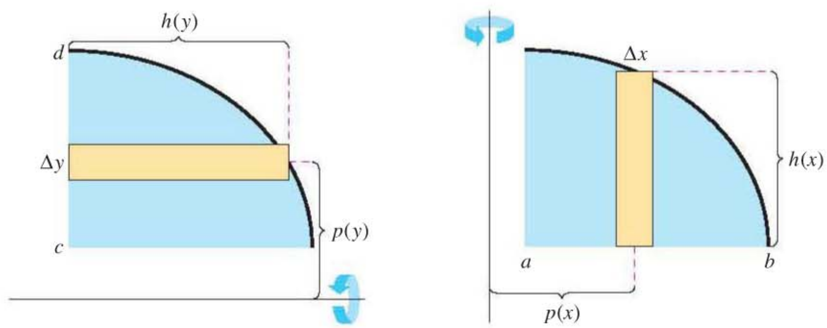


Figure 7.28

THE SHELL METHOD

To find the volume of a solid of revolution with the **shell method,** use one of the following, as shown in Figure 7.29.

Horizontal Axis of Revolution	*Vertical Axis of Revolution*
Volume $= V = 2\pi \int_c^d p(y)h(y)\,dy$	Volume $= V = 2\pi \int_a^b p(x)h(x)\,dx$



Horizontal axis of revolution Vertical axis of revolution
Figure 7.29

EXAMPLE 1 Using the Shell Method to Find Volume

Find the volume of the solid of revolution formed by revolving the region bounded by

$$y = x - x^3$$

and the x-axis $(0 \le x \le 1)$ about the y-axis.

Solution Because the axis of revolution is vertical, use a vertical representative rectangle, as shown in Figure 7.30. The width Δx indicates that x is the variable of integration. The distance from the center of the rectangle to the axis of revolution is $p(x) = x$, and the height of the rectangle is

$$h(x) = x - x^3.$$

Because x ranges from 0 to 1, the volume of the solid is

$$V = 2\pi \int_a^b p(x)h(x)\,dx = 2\pi \int_0^1 x(x - x^3)\,dx \qquad \text{Apply shell method.}$$

$$= 2\pi \int_0^1 (-x^4 + x^2)\,dx \qquad \text{Simplify.}$$

$$= 2\pi \left[-\frac{x^5}{5} + \frac{x^3}{3} \right]_0^1 \qquad \text{Integrate.}$$

$$= 2\pi \left(-\frac{1}{5} + \frac{1}{3} \right)$$

$$= \frac{4\pi}{15}.$$

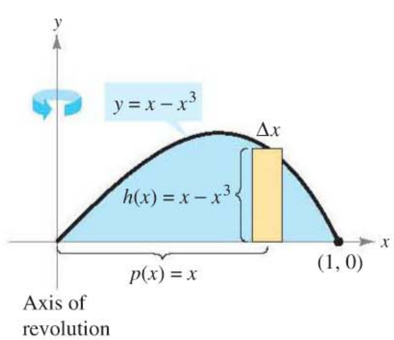


Figure 7.30

EXAMPLE 2 Using the Shell Method to Find Volume

Find the volume of the solid of revolution formed by revolving the region bounded by the graph of

$$x = e^{-y^2}$$

and the y-axis $(0 \le y \le 1)$ about the x-axis.

Solution Because the axis of revolution is horizontal, use a horizontal representative rectangle, as shown in Figure 7.31. The width Δy indicates that y is the variable of integration. The distance from the center of the rectangle to the axis of revolution is $p(y) = y$, and the height of the rectangle is $h(y) = e^{-y^2}$. Because y ranges from 0 to 1, the volume of the solid is

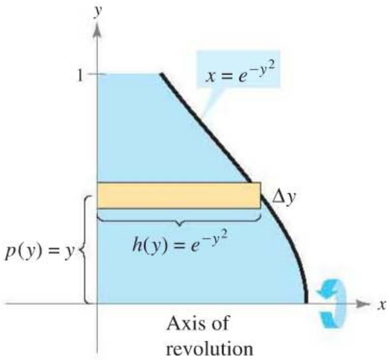


Figure 7.31

$$V = 2\pi \int_c^d p(y)h(y)\,dy = 2\pi \int_0^1 ye^{-y^2}\,dy \qquad \text{Apply shell method.}$$

$$= -\pi\Big[e^{-y^2}\Big]_0^1 \qquad \text{Integrate.}$$

$$= \pi\left(1 - \frac{1}{e}\right)$$

$$\approx 1.986.$$

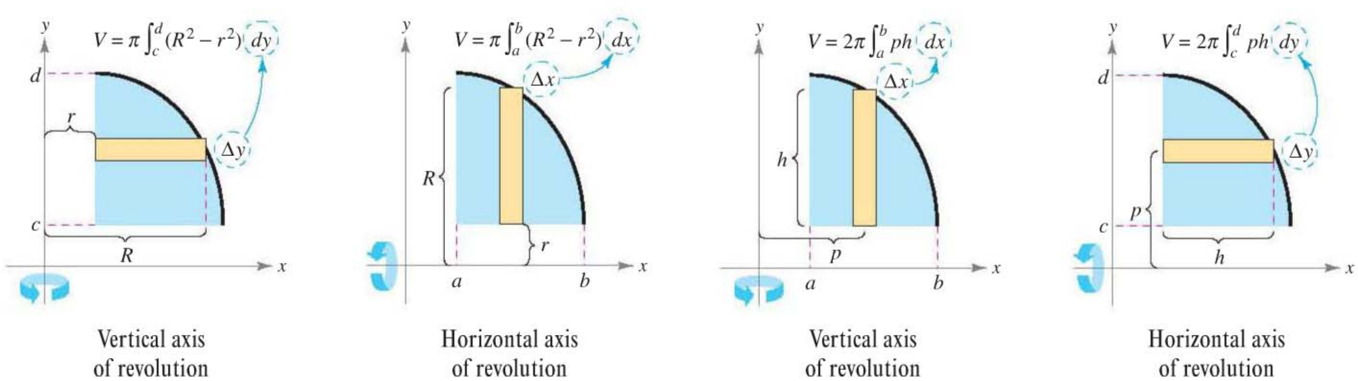
NOTE To see the advantage of using the shell method in Example 2, solve the equation $x = e^{-y^2}$ for y.

$$y = \begin{cases} 1, & 0 \le x \le 1/e \\ \sqrt{-\ln x}, & 1/e < x \le 1 \end{cases}$$

Then use this equation to find the volume using the disk method.

Comparison of Disk and Shell Methods

The disk and shell methods can be distinguished as follows. For the disk method, the representative rectangle is always *perpendicular* to the axis of revolution, whereas for the shell method, the representative rectangle is always *parallel* to the axis of revolution, as shown in Figure 7.32.



Disk method: Representative rectangle is perpendicular to the axis of revolution.

Shell method: Representative rectangle is parallel to the axis of revolution.

Figure 7.32

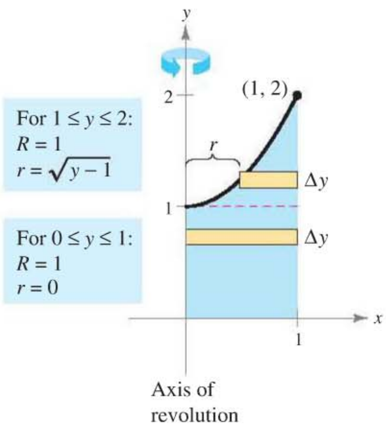
Often, one method is more convenient to use than the other. The following example illustrates a case in which the shell method is preferable.

EXAMPLE 3 Shell Method Preferable

Find the volume of the solid formed by revolving the region bounded by the graphs of

$$y = x^2 + 1, \quad y = 0, \quad x = 0, \quad \text{and} \quad x = 1$$

about the y-axis.



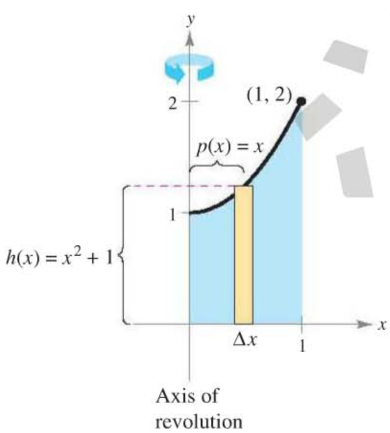
(a) Disk method

Solution In Example 4 in the preceding section, you saw that the washer method requires two integrals to determine the volume of this solid. See Figure 7.33(a).

$$\begin{aligned}
V &= \pi \int_0^1 (1^2 - 0^2)\,dy + \pi \int_1^2 \left[1^2 - \left(\sqrt{y-1}\right)^2\right] dy && \text{Apply washer method.}\\
&= \pi \int_0^1 1\,dy + \pi \int_1^2 (2 - y)\,dy && \text{Simplify.}\\
&= \pi \Big[y\Big]_0^1 + \pi \left[2y - \frac{y^2}{2}\right]_1^2 && \text{Integrate.}\\
&= \pi + \pi\left(4 - 2 - 2 + \frac{1}{2}\right)\\
&= \frac{3\pi}{2}
\end{aligned}$$

In Figure 7.33(b), you can see that the shell method requires only one integral to find the volume.

$$\begin{aligned}
V &= 2\pi \int_a^b p(x)h(x)\,dx && \text{Apply shell method.}\\
&= 2\pi \int_0^1 x(x^2 + 1)\,dx\\
&= 2\pi \left[\frac{x^4}{4} + \frac{x^2}{2}\right]_0^1 && \text{Integrate.}\\
&= 2\pi\left(\frac{3}{4}\right)\\
&= \frac{3\pi}{2}
\end{aligned}$$



(b) Shell method

Figure 7.33

Suppose the region in Example 3 were revolved about the vertical line $x = 1$. Would the resulting solid of revolution have a greater volume or a smaller volume than the solid in Example 3? Without integrating, you should be able to reason that the resulting solid would have a smaller volume because "more" of the revolved region would be closer to the axis of revolution. To confirm this, try solving the following integral, which gives the volume of the solid.

$$V = 2\pi \int_0^1 (1 - x)(x^2 + 1)\,dx \qquad p(x) = 1 - x$$

FOR FURTHER INFORMATION To learn more about the disk and shell methods, see the article "The Disk and Shell Method" by Charles A. Cable in *The American Mathematical Monthly*. To view this article, go to the website *www.matharticles.com*.

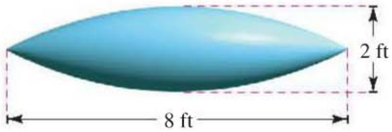


Figure 7.34

EXAMPLE 4 Volume of a Pontoon

A pontoon is to be made in the shape shown in Figure 7.34. The pontoon is designed by rotating the graph of

$$y = 1 - \frac{x^2}{16}, \quad -4 \le x \le 4$$

about the x-axis, where x and y are measured in feet. Find the volume of the pontoon.

Solution Refer to Figure 7.35(a) and use the disk method as follows.

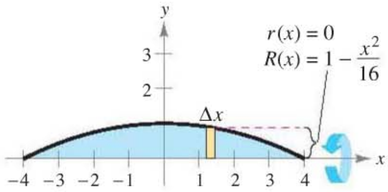
$$V = \pi \int_{-4}^{4} \left(1 - \frac{x^2}{16}\right)^2 dx \qquad \text{Apply disk method.}$$

$$= \pi \int_{-4}^{4} \left(1 - \frac{x^2}{8} + \frac{x^4}{256}\right) dx \qquad \text{Simplify.}$$

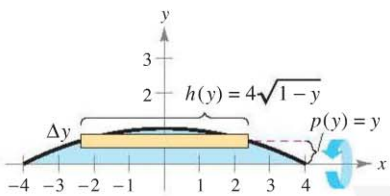
$$= \pi \left[x - \frac{x^3}{24} + \frac{x^5}{1280}\right]_{-4}^{4} \qquad \text{Integrate.}$$

$$= \frac{64\pi}{15} \approx 13.4 \text{ cubic feet}$$

Try using Figure 7.35(b) to set up the integral for the volume using the shell method. Does the integral seem more complicated?



(a) Disk method



(b) Shell method

Figure 7.35

To use the shell method in Example 4, you would have to solve for x in terms of y in the equation

$$y = 1 - (x^2/16).$$

Sometimes, solving for x is very difficult (or even impossible). In such cases you must use a vertical rectangle (of width Δx), thus making x the variable of integration. The position (horizontal or vertical) of the axis of revolution then determines the method to be used. This is shown in Example 5.

EXAMPLE 5 Shell Method Necessary

Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^3 + x + 1$, $y = 1$, and $x = 1$ about the line $x = 2$, as shown in Figure 7.36.

Solution In the equation $y = x^3 + x + 1$, you cannot easily solve for x in terms of y. (See Section 3.8 on Newton's Method.) Therefore, the variable of integration must be x, and you should choose a vertical representative rectangle. Because the rectangle is parallel to the axis of revolution, use the shell method and obtain

$$V = 2\pi \int_a^b p(x)h(x)\,dx = 2\pi \int_0^1 (2 - x)(x^3 + x + 1 - 1)\,dx \qquad \text{Apply shell method.}$$

$$= 2\pi \int_0^1 (-x^4 + 2x^3 - x^2 + 2x)\,dx \qquad \text{Simplify.}$$

$$= 2\pi \left[-\frac{x^5}{5} + \frac{x^4}{2} - \frac{x^3}{3} + x^2\right]_0^1 \qquad \text{Integrate.}$$

$$= 2\pi\left(-\frac{1}{5} + \frac{1}{2} - \frac{1}{3} + 1\right)$$

$$= \frac{29\pi}{15}.$$

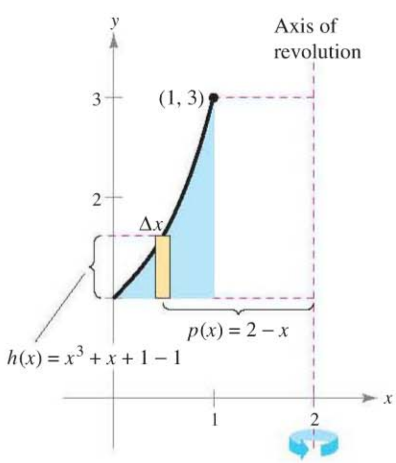


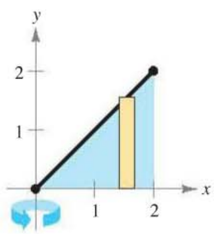
Figure 7.36

7.3 Exercises

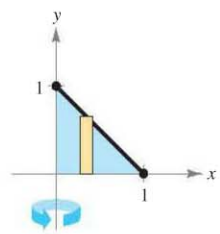
See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1–14, use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the y-axis.

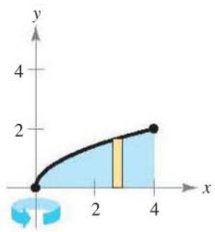
1. $y = x$



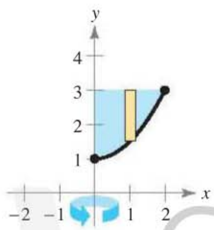
2. $y = 1 - x$



3. $y = \sqrt{x}$



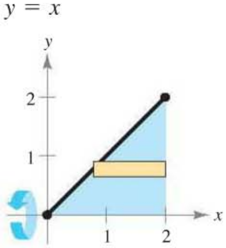
4. $y = \frac{1}{2}x^2 + 1$



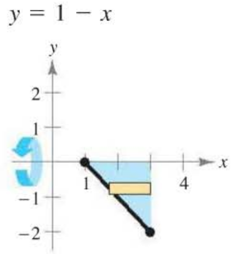
5. $y = x^2, \quad y = 0, \quad x = 3$
6. $y = \frac{1}{4}x^2, \quad y = 0, \quad x = 6$
7. $y = x^2, \quad y = 4x - x^2$
8. $y = 4 - x^2, \quad y = 0$
9. $y = 4x - x^2, \quad x = 0, \quad y = 4$
10. $y = 3x, \quad y = 6, \quad x = 0$
11. $y = \sqrt{x - 2}, \quad y = 0, \quad x = 4$
12. $y = -x^2 + 1, \quad y = 0$
13. $y = \dfrac{1}{\sqrt{2\pi}} e^{-x^2/2}, \quad y = 0, \quad x = 0, \quad x = 1$
14. $y = \begin{cases} \dfrac{\sin x}{x}, & x > 0 \\ 1, & x = 0 \end{cases}, \quad y = 0, \quad x = 0, \quad x = \pi$

In Exercises 15–22, use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the x-axis.

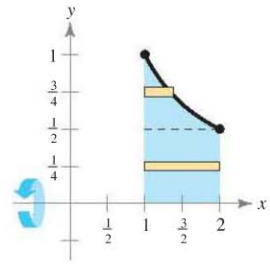
15. $y = x$



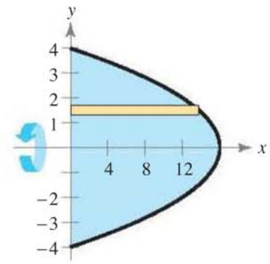
16. $y = 1 - x$



17. $y = \dfrac{1}{x}$



18. $x + y^2 = 16$



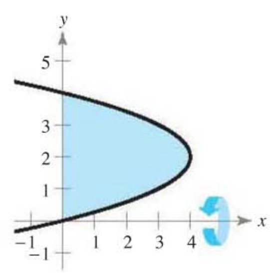
19. $y = x^3, \quad x = 0, \quad y = 8$
20. $y = x^2, \quad x = 0, \quad y = 9$
21. $x + y = 4, \quad y = x, \quad y = 0$
22. $y = \sqrt{x + 2}, \quad y = x, \quad y = 0$

In Exercises 23–26, use the shell method to find the volume of the solid generated by revolving the plane region about the given line.

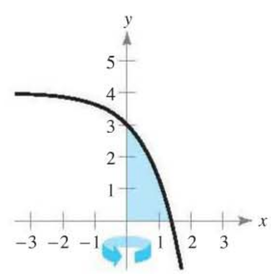
23. $y = 4x - x^2, \quad y = 0,$ about the line $x = 5$
24. $y = \sqrt{x}, \quad y = 0, \quad x = 4,$ about the line $x = 6$
25. $y = x^2, \quad y = 4x - x^2,$ about the line $x = 4$
26. $y = x^2, \quad y = 4x - x^2,$ about the line $x = 2$

In Exercises 27 and 28, decide whether it is more convenient to use the disk method or the shell method to find the volume of the solid of revolution. Explain your reasoning. (Do not find the volume.)

27. $(y - 2)^2 = 4 - x$



28. $y = 4 - e^x$



In Exercises 29–32, use the disk *or* the shell method to find the volume of the solid generated by revolving the region bounded by the graphs of the equations about each given line.

29. $y = x^3, \quad y = 0, \quad x = 2$
   (a) the x-axis (b) the y-axis (c) the line $x = 4$
30. $y = \dfrac{10}{x^2}, \quad y = 0, \quad x = 1, \quad x = 5$
   (a) the x-axis (b) the y-axis (c) the line $y = 10$
31. $x^{1/2} + y^{1/2} = a^{1/2}, \quad x = 0, \quad y = 0$
   (a) the x-axis (b) the y-axis (c) the line $x = a$

32. $x^{2/3} + y^{2/3} = a^{2/3}$, $a > 0$ (hypocycloid)

(a) the x-axis (b) the y-axis

In Exercises 33–36, (a) use a graphing utility to graph the plane region bounded by the graphs of the equations, and (b) use the integration capabilities of the graphing utility to approximate the volume of the solid generated by revolving the region about the y-axis.

33. $x^{4/3} + y^{4/3} = 1$, $x = 0$, $y = 0$, first quadrant

34. $y = \sqrt{1 - x^3}$, $y = 0$, $x = 0$

35. $y = \sqrt[3]{(x-2)^2(x-6)^2}$, $y = 0$, $x = 2$, $x = 6$

36. $y = \dfrac{2}{1 + e^{1/x}}$, $y = 0$, $x = 1$, $x = 3$

Think About It **In Exercises 37 and 38, determine which value best approximates the volume of the solid generated by revolving the region bounded by the graphs of the equations about the y-axis. (Make your selection on the basis of a sketch of the solid and *not* by performing any calculations.)**

37. $y = 2e^{-x}$, $y = 0$, $x = 0$, $x = 2$

(a) $\frac{3}{2}$ (b) -2 (c) 4 (d) 7.5 (e) 15

38. $y = \tan x$, $y = 0$, $x = 0$, $x = \dfrac{\pi}{4}$

(a) 3.5 (b) $-\frac{9}{4}$ (c) 8 (d) 10 (e) 1

WRITING ABOUT CONCEPTS

39. The region in the figure is revolved about the indicated axes and line. Order the volumes of the resulting solids from least to greatest. Explain your reasoning.

(a) x-axis (b) y-axis (c) $x = 4$

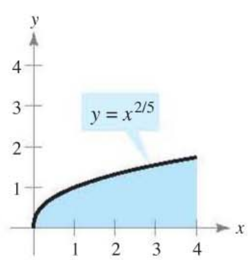


Figure for 39

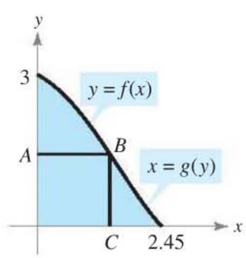


Figure for 40

40. (a) Describe the figure generated by revolving segment AB about the y-axis (see figure).

(b) Describe the figure generated by revolving segment BC about the y-axis.

(c) Assume the curve in the figure can be described as $y = f(x)$ or $x = g(y)$. A solid is generated by revolving the region bounded by the curve, $y = 0$, and $x = 0$ about the y-axis. Set up integrals to find the volume of this solid using the disk method and the shell method. (Do not integrate.)

WRITING ABOUT CONCEPTS (continued)

In Exercises 41 and 42, give a geometric argument that explains why the integrals have equal values.

41. $\pi\int_1^5 (x - 1)\,dx = 2\pi\int_0^2 y[5 - (y^2 + 1)]\,dy$

42. $\pi\int_0^2 [16 - (2y)^2]\,dy = 2\pi\int_0^4 x\left(\dfrac{x}{2}\right)dx$

43. Consider a solid that is generated by revolving a plane region about the y-axis. Describe the position of a representative rectangle when using (a) the shell method and (b) the disk method to find the volume of the solid.

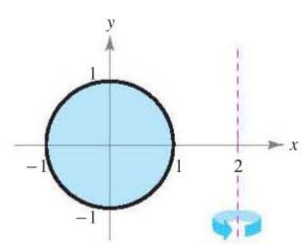
CAPSTONE

44. Consider the plane region bounded by the graphs of $y = k$, $y = 0$, $x = 0$, and $x = b$, where $k > 0$ and $b > 0$. What are the heights and radii of the cylinders generated when this region is revolved about (a) the x-axis and (b) the y-axis?

45. ***Machine Part*** A solid is generated by revolving the region bounded by $y = \frac{1}{2}x^2$ and $y = 2$ about the y-axis. A hole, centered along the axis of revolution, is drilled through this solid so that one-fourth of the volume is removed. Find the diameter of the hole.

46. ***Machine Part*** A solid is generated by revolving the region bounded by $y = \sqrt{9 - x^2}$ and $y = 0$ about the y-axis. A hole, centered along the axis of revolution, is drilled through this solid so that one-third of the volume is removed. Find the diameter of the hole.

47. ***Volume of a Torus*** A torus is formed by revolving the region bounded by the circle $x^2 + y^2 = 1$ about the line $x = 2$ (see figure). Find the volume of this "doughnut-shaped" solid. (*Hint:* The integral $\int_{-1}^{1} \sqrt{1 - x^2}\,dx$ represents the area of a semicircle.)



48. ***Volume of a Torus*** Repeat Exercise 47 for a torus formed by revolving the region bounded by the circle $x^2 + y^2 = r^2$ about the line $x = R$, where $r < R$.

In Exercises 49–52, the integral represents the volume of a solid of revolution. Identify (a) the plane region that is revolved and (b) the axis of revolution.

49. $2\pi\int_0^2 x^3\,dx$

50. $2\pi\int_0^1 y - y^{3/2}\,dy$

51. $2\pi\int_0^6 (y + 2)\sqrt{6 - y}\,dy$

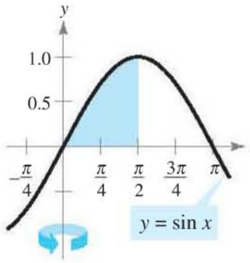
52. $2\pi\int_0^1 (4 - x)e^x\,dx$

53. (a) Use differentiation to verify that

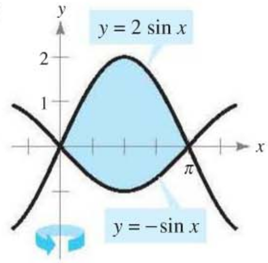
$$\int x \sin x \, dx = \sin x - x \cos x + C.$$

(b) Use the result of part (a) to find the volume of the solid generated by revolving each plane region about the y-axis.

(i)



(ii)

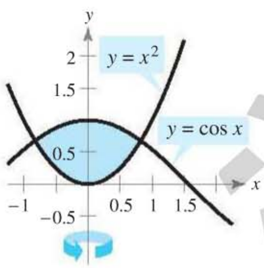


54. (a) Use differentiation to verify that

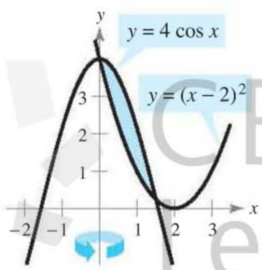
$$\int x \cos x \, dx = \cos x + x \sin x + C.$$

(b) Use the result of part (a) to find the volume of the solid generated by revolving each plane region about the y-axis. (*Hint:* Begin by approximating the points of intersection.)

(i)



(ii)



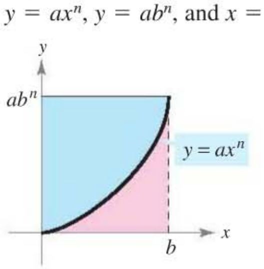
55. ***Volume of a Segment of a Sphere*** Let a sphere of radius r be cut by a plane, thereby forming a segment of height h. Show that the volume of this segment is $\frac{1}{3}\pi h^2(3r - h)$.

56. ***Volume of an Ellipsoid*** Consider the plane region bounded by the graph of

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

where $a > 0$ and $b > 0$. Show that the volume of the ellipsoid formed when this region is revolved about the y-axis is $\frac{4}{3}\pi a^2 b$. What is the volume when the region is revolved about the x-axis?

57. ***Exploration*** Consider the region bounded by the graphs of $y = ax^n$, $y = ab^n$, and $x = 0$ (see figure).



(a) Find the ratio $R_1(n)$ of the area of the region to the area of the circumscribed rectangle.

(b) Find $\lim_{n\to\infty} R_1(n)$ and compare the result with the area of the circumscribed rectangle.

(c) Find the volume of the solid of revolution formed by revolving the region about the y-axis. Find the ratio $R_2(n)$ of this volume to the volume of the circumscribed right circular cylinder.

(d) Find $\lim_{n\to\infty} R_2(n)$ and compare the result with the volume of the circumscribed cylinder.

(e) Use the results of parts (b) and (d) to make a conjecture about the shape of the graph of $y = ax^n$ $(0 \le x \le b)$ as $n \to \infty$.

58. ***Think About It*** Match each integral with the solid whose volume it represents, and give the dimensions of each solid.

(a) Right circular cone (b) Torus (c) Sphere

(d) Right circular cylinder (e) Ellipsoid

(i) $2\pi \int_0^r hx \, dx$ (ii) $2\pi \int_0^r hx\left(1 - \frac{x}{r}\right) dx$

(iii) $2\pi \int_0^r 2x\sqrt{r^2 - x^2} \, dx$ (iv) $2\pi \int_0^b 2ax \sqrt{1 - \frac{x^2}{b^2}} \, dx$

(v) $2\pi \int_{-r}^r (R - x)\left(2\sqrt{r^2 - x^2}\right) dx$

59. ***Volume of a Storage Shed*** A storage shed has a circular base of diameter 80 feet. Starting at the center, the interior height is measured every 10 feet and recorded in the table (see figure).

x	0	10	20	30	40
Height	50	45	40	20	0

(a) Use Simpson's Rule to approximate the volume of the shed.

(b) Note that the roof line consists of two line segments. Find the equations of the line segments and use integration to find the volume of the shed.

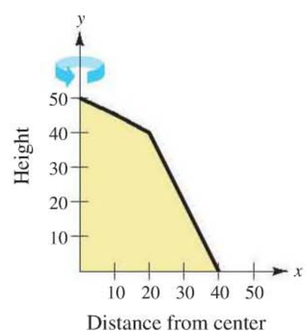


Figure for 59

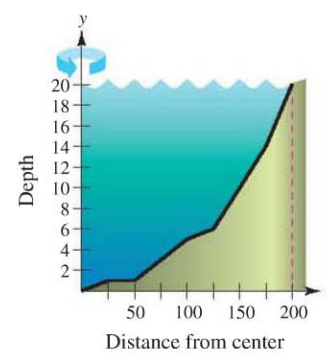


Figure for 60

60. ***Modeling Data*** A pond is approximately circular, with a diameter of 400 feet. Starting at the center, the depth of the water is measured every 25 feet and recorded in the table (see figure).

x	0	25	50	75	100	125	150	175	200
Depth	20	19	19	17	15	14	10	6	0

(a) Use Simpson's Rule to approximate the volume of water in the pond.

(b) Use the regression capabilities of a graphing utility to find a quadratic model for the depths recorded in the table. Use the graphing utility to plot the depths and graph the model.

(c) Use the integration capabilities of a graphing utility and the model in part (b) to approximate the volume of water in the pond.

(d) Use the result of part (c) to approximate the number of gallons of water in the pond if 1 cubic foot of water is approximately 7.48 gallons.

61. Let V_1 and V_2 be the volumes of the solids that result when the plane region bounded by $y = 1/x$, $y = 0$, $x = \frac{1}{4}$, and $x = c$ (where $c > \frac{1}{4}$) is revolved about the x-axis and the y-axis, respectively. Find the value of c for which $V_1 = V_2$.

62. The region bounded by $y = r^2 - x^2$, $y = 0$, and $x = 0$ is revolved about the y-axis to form a paraboloid. A hole, centered along the axis of revolution, is drilled through this solid. The hole has a radius k, $0 < k < r$. Find the volume of the resulting ring (a) by integrating with respect to x and (b) by integrating with respect to y.

63. Consider the graph of $y^2 = x(4 - x)^2$ (see figure). Find the volumes of the solids that are generated when the loop of this graph is revolved about (a) the x-axis, (b) the y-axis, and (c) the line $x = 4$.

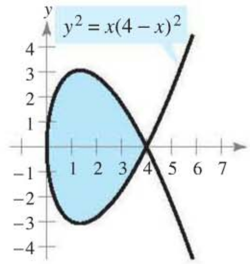


Figure for 63

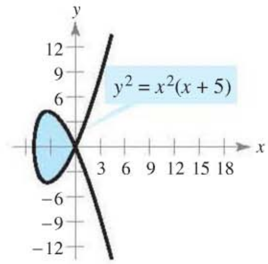


Figure for 64

64. Consider the graph of $y^2 = x^2(x + 5)$ (see figure). Find the volumes of the solids that are generated when the loop of this graph is revolved about (a) the x-axis, (b) the y-axis, and (c) the line $x = -5$.

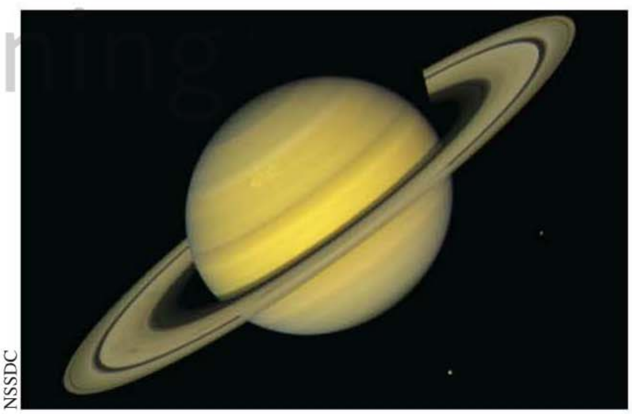
SECTION PROJECT

Saturn

The Oblateness of Saturn Saturn is the most oblate of the nine planets in our solar system. Its equatorial radius is 60,268 kilometers and its polar radius is 54,364 kilometers. The color-enhanced photograph of Saturn was taken by Voyager 1. In the photograph, the oblateness of Saturn is clearly visible.

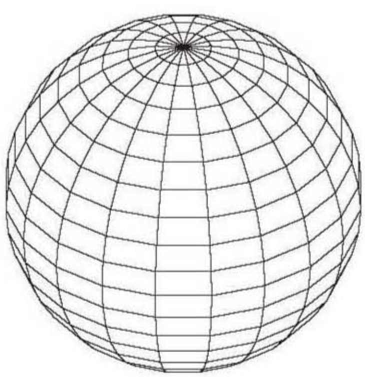
(a) Find the ratio of the volumes of the sphere and the oblate ellipsoid shown below.

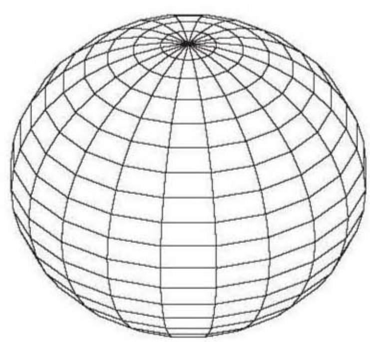
(b) If a planet were spherical and had the same volume as Saturn, what would its radius be?


NSSDC

Computer model of "spherical Saturn," whose equatorial radius is equal to its polar radius. The equation of the cross section passing through the pole is

$x^2 + y^2 = 60{,}268^2.$





Computer model of "oblate Saturn," whose equatorial radius is greater than its polar radius. The equation of the cross section passing through the pole is

$$\frac{x^2}{60{,}268^2} + \frac{y^2}{54{,}364^2} = 1.$$