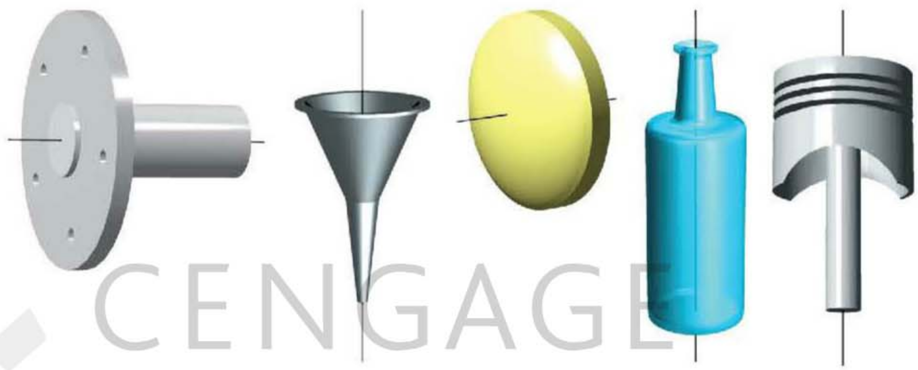
7.2 Volume: The Disk Method

- Find the volume of a solid of revolution using the disk method.
- Find the volume of a solid of revolution using the washer method.
- Find the volume of a solid with known cross sections.

The Disk Method

You have already learned that area is only one of the *many* applications of the definite integral. Another important application is its use in finding the volume of a three-dimensional solid. In this section you will study a particular type of three-dimensional solid—one whose cross sections are similar. Solids of revolution are used commonly in engineering and manufacturing. Some examples are axles, funnels, pills, bottles, and pistons, as shown in Figure 7.12.

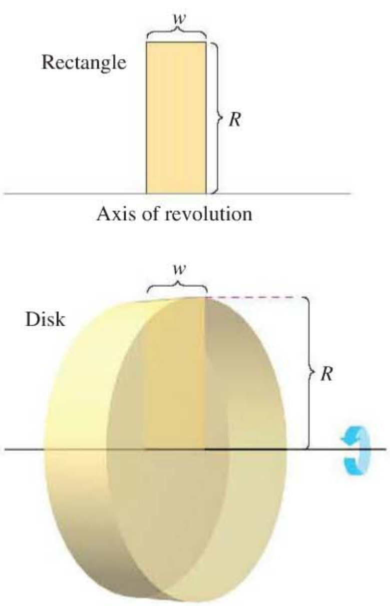


Solids of revolution
Figure 7.12

If a region in the plane is revolved about a line, the resulting solid is a **solid of revolution,** and the line is called the **axis of revolution.** The simplest such solid is a right circular cylinder or **disk,** which is formed by revolving a rectangle about an axis adjacent to one side of the rectangle, as shown in Figure 7.13. The volume of such a disk is

$$\text{Volume of disk} = (\text{area of disk})(\text{width of disk})$$
$$= \pi R^2 w$$

where R is the radius of the disk and w is the width.



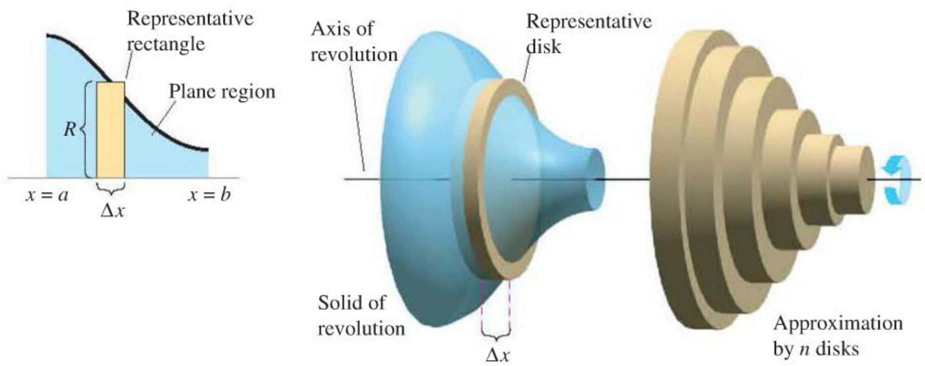
Volume of a disk: $\pi R^2 w$
Figure 7.13

To see how to use the volume of a disk to find the volume of a general solid of revolution, consider a solid of revolution formed by revolving the plane region in Figure 7.14 about the indicated axis. To determine the volume of this solid, consider a representative rectangle in the plane region. When this rectangle is revolved about the axis of revolution, it generates a representative disk whose volume is

$$\Delta V = \pi R^2 \Delta x.$$

Approximating the volume of the solid by n such disks of width Δx and radius $R(x_i)$ produces

$$\text{Volume of solid} \approx \sum_{i=1}^{n} \pi [R(x_i)]^2 \, \Delta x$$
$$= \pi \sum_{i=1}^{n} [R(x_i)]^2 \, \Delta x.$$



Disk method
Figure 7.14

This approximation appears to become better and better as $\|\Delta\| \to 0$ ($n \to \infty$). So, you can define the volume of the solid as

$$\text{Volume of solid} = \lim_{\|\Delta\| \to 0} \pi \sum_{i=1}^{n} [R(x_i)]^2 \, \Delta x = \pi \int_a^b [R(x)]^2 \, dx.$$

Schematically, the disk method looks like this.

Known Precalculus Formula		*Representative Element*		*New Integration Formula*
Volume of disk $V = \pi R^2 w$	⇨	$\Delta V = \pi [R(x_i)]^2 \, \Delta x$	⇨	Solid of revolution $V = \pi \int_a^b [R(x)]^2 \, dx$

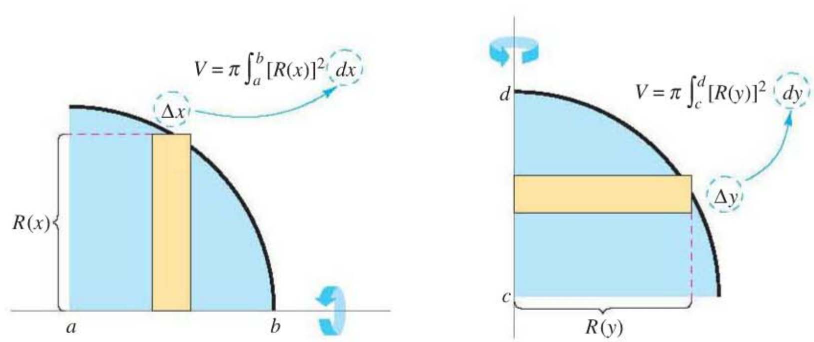
A similar formula can be derived if the axis of revolution is vertical.

THE DISK METHOD

To find the volume of a solid of revolution with the **disk method,** use one of the following, as shown in Figure 7.15.

Horizontal Axis of Revolution	*Vertical Axis of Revolution*
$\text{Volume} = V = \pi \int_a^b [R(x)]^2 \, dx$	$\text{Volume} = V = \pi \int_c^d [R(y)]^2 \, dy$

NOTE In Figure 7.15, note that you can determine the variable of integration by placing a representative rectangle in the plane region "perpendicular" to the axis of revolution. If the width of the rectangle is Δx, integrate with respect to x, and if the width of the rectangle is Δy, integrate with respect to y.



Horizontal axis of revolution　　Vertical axis of revolution
Figure 7.15

The simplest application of the disk method involves a plane region bounded by the graph of f and the x-axis. If the axis of revolution is the x-axis, the radius $R(x)$ is simply $f(x)$.

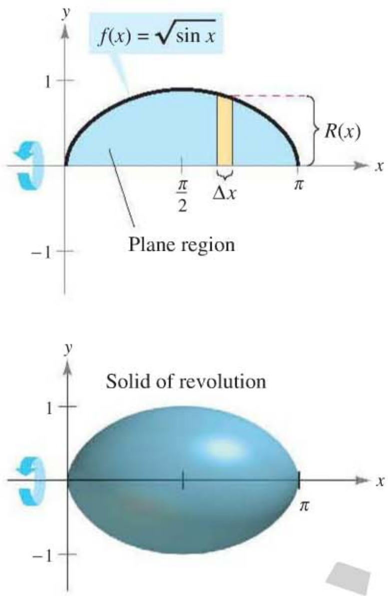


Figure 7.16

EXAMPLE 1 Using the Disk Method

Find the volume of the solid formed by revolving the region bounded by the graph of

$$f(x) = \sqrt{\sin x}$$

and the x-axis $(0 \le x \le \pi)$ about the x-axis.

Solution From the representative rectangle in the upper graph in Figure 7.16, you can see that the radius of this solid is

$$\begin{aligned} R(x) &= f(x) \\ &= \sqrt{\sin x}. \end{aligned}$$

So, the volume of the solid of revolution is

$$\begin{aligned} V = \pi\int_a^b [R(x)]^2\,dx &= \pi\int_0^{\pi} \left(\sqrt{\sin x}\right)^2 dx && \text{Apply disk method.} \\ &= \pi\int_0^{\pi} \sin x\,dx && \text{Simplify.} \\ &= \pi\Big[-\cos x\Big]_0^{\pi} && \text{Integrate.} \\ &= \pi(1+1) \\ &= 2\pi. \end{aligned}$$

EXAMPLE 2 Revolving About a Line That Is Not a Coordinate Axis

Find the volume of the solid formed by revolving the region bounded by

$$f(x) = 2 - x^2$$

and $g(x) = 1$ about the line $y = 1$, as shown in Figure 7.17.

Solution By equating $f(x)$ and $g(x)$, you can determine that the two graphs intersect when $x = \pm 1$. To find the radius, subtract $g(x)$ from $f(x)$.

$$\begin{aligned} R(x) &= f(x) - g(x) \\ &= (2 - x^2) - 1 \\ &= 1 - x^2 \end{aligned}$$

Finally, integrate between -1 and 1 to find the volume.

$$\begin{aligned} V = \pi\int_a^b [R(x)]^2\,dx &= \pi\int_{-1}^{1} (1 - x^2)^2\,dx && \text{Apply disk method.} \\ &= \pi\int_{-1}^{1} (1 - 2x^2 + x^4)\,dx && \text{Simplify.} \\ &= \pi\left[x - \frac{2x^3}{3} + \frac{x^5}{5}\right]_{-1}^{1} && \text{Integrate.} \\ &= \frac{16\pi}{15} \end{aligned}$$

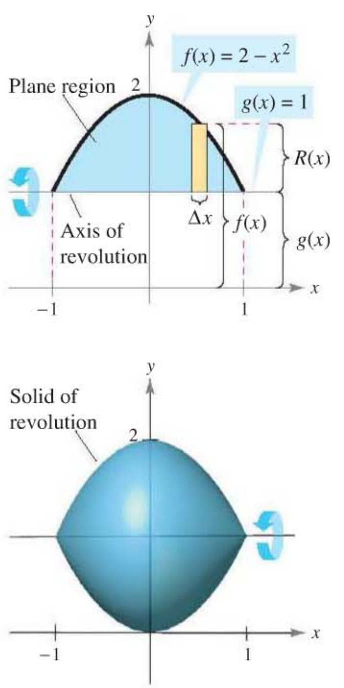


Figure 7.17

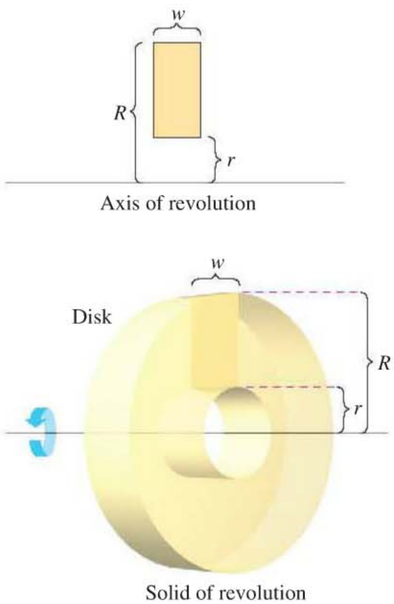


Figure 7.18

The Washer Method

The disk method can be extended to cover solids of revolution with holes by replacing the representative disk with a representative **washer.** The washer is formed by revolving a rectangle about an axis, as shown in Figure 7.18. If r and R are the inner and outer radii of the washer and w is the width of the washer, the volume is given by

$$\text{Volume of washer} = \pi(R^2 - r^2)w.$$

To see how this concept can be used to find the volume of a solid of revolution, consider a region bounded by an **outer radius** $R(x)$ and an **inner radius** $r(x)$, as shown in Figure 7.19. If the region is revolved about its axis of revolution, the volume of the resulting solid is given by

$$V = \pi\int_a^b ([R(x)]^2 - [r(x)]^2)\,dx. \qquad \text{Washer method}$$

Note that the integral involving the inner radius represents the volume of the hole and is *subtracted* from the integral involving the outer radius.

Figure 7.19

EXAMPLE 3 Using the Washer Method

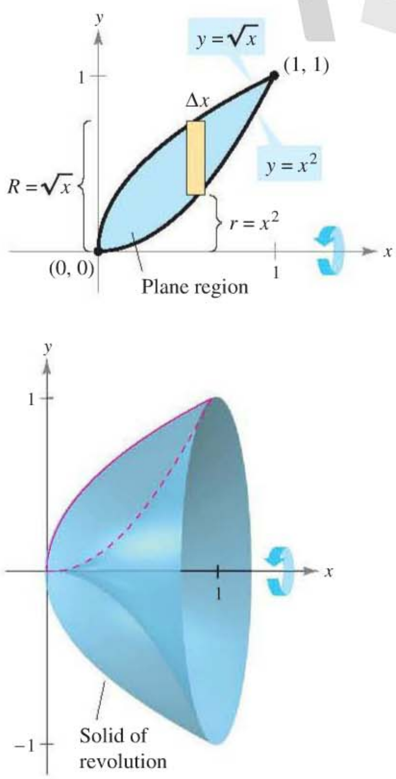
Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$ about the x-axis, as shown in Figure 7.20.

Solution In Figure 7.20, you can see that the outer and inner radii are as follows.

$$R(x) = \sqrt{x} \qquad \text{Outer radius}$$
$$r(x) = x^2 \qquad \text{Inner radius}$$

Integrating between 0 and 1 produces

$$V = \pi\int_a^b ([R(x)]^2 - [r(x)]^2)\,dx \qquad \text{Apply washer method.}$$
$$= \pi\int_0^1 \left[\left(\sqrt{x}\right)^2 - (x^2)^2\right]dx$$
$$= \pi\int_0^1 (x - x^4)\,dx \qquad \text{Simplify.}$$
$$= \pi\left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1 \qquad \text{Integrate.}$$
$$= \frac{3\pi}{10}.$$



Solid of revolution
Figure 7.20

In each example so far, the axis of revolution has been *horizontal* and you have integrated with respect to x. In the next example, the axis of revolution is *vertical* and you integrate with respect to y. In this example, you need two separate integrals to compute the volume.

EXAMPLE 4 Integrating with Respect to y, Two-Integral Case

Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^2 + 1$, $y = 0$, $x = 0$, and $x = 1$ about the y-axis, as shown in Figure 7.21.

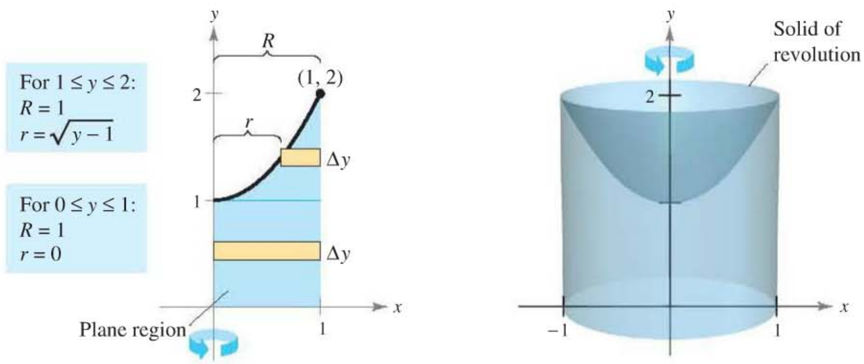


Figure 7.21

Solution For the region shown in Figure 7.21, the outer radius is simply $R = 1$. There is, however, no convenient formula that represents the inner radius. When $0 \le y \le 1$, $r = 0$, but when $1 \le y \le 2$, r is determined by the equation $y = x^2 + 1$, which implies that $r = \sqrt{y - 1}$.

$$r(y) = \begin{cases} 0, & 0 \le y \le 1 \\ \sqrt{y-1}, & 1 \le y \le 2 \end{cases}$$

Using this definition of the inner radius, you can use two integrals to find the volume.

$$\begin{aligned} V &= \pi \int_0^1 (1^2 - 0^2)\,dy + \pi \int_1^2 \left[1^2 - \left(\sqrt{y-1}\right)^2\right] dy && \text{Apply washer method.} \\ &= \pi \int_0^1 1\,dy + \pi \int_1^2 (2 - y)\,dy && \text{Simplify.} \\ &= \pi \Big[y\Big]_0^1 + \pi \left[2y - \frac{y^2}{2}\right]_1^2 && \text{Integrate.} \\ &= \pi + \pi\left(4 - 2 - 2 + \frac{1}{2}\right) = \frac{3\pi}{2} \end{aligned}$$

Note that the first integral $\pi \int_0^1 1\,dy$ represents the volume of a right circular cylinder of radius 1 and height 1. This portion of the volume could have been determined without using calculus. ■

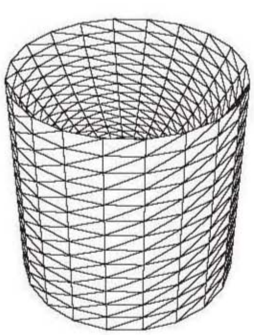
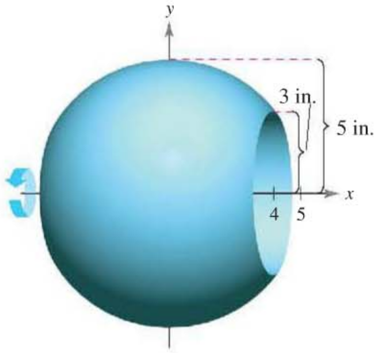


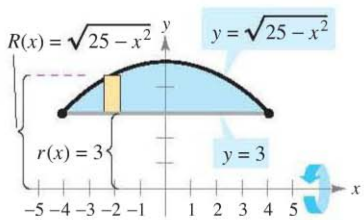
Figure 7.22

TECHNOLOGY Some graphing utilities have the capability of generating (or have built-in software capable of generating) a solid of revolution. If you have access to such a utility, use it to graph some of the solids of revolution described in this section. For instance, the solid in Example 4 might appear like that shown in Figure 7.22.



Solid of revolution

(a)



Plane region

(b)

Figure 7.23

EXAMPLE 5 Manufacturing

A manufacturer drills a hole through the center of a metal sphere of radius 5 inches, as shown in Figure 7.23(a). The hole has a radius of 3 inches. What is the volume of the resulting metal ring?

Solution You can imagine the ring to be generated by a segment of the circle whose equation is $x^2 + y^2 = 25$, as shown in Figure 7.23(b). Because the radius of the hole is 3 inches, you can let $y = 3$ and solve the equation $x^2 + y^2 = 25$ to determine that the limits of integration are $x = \pm 4$. So, the inner and outer radii are $r(x) = 3$ and $R(x) = \sqrt{25 - x^2}$ and the volume is given by

$$\begin{aligned} V = \pi\int_a^b ([R(x)]^2 - [r(x)]^2)\,dx &= \pi\int_{-4}^{4} \left[\left(\sqrt{25 - x^2}\right)^2 - (3)^2\right] dx \\ &= \pi\int_{-4}^{4} (16 - x^2)\,dx \\ &= \pi\left[16x - \frac{x^3}{3}\right]_{-4}^{4} \\ &= \frac{256\pi}{3} \text{ cubic inches.} \end{aligned}$$

Solids with Known Cross Sections

With the disk method, you can find the volume of a solid having a circular cross section whose area is $A = \pi R^2$. This method can be generalized to solids of any shape, as long as you know a formula for the area of an arbitrary cross section. Some common cross sections are squares, rectangles, triangles, semicircles, and trapezoids.

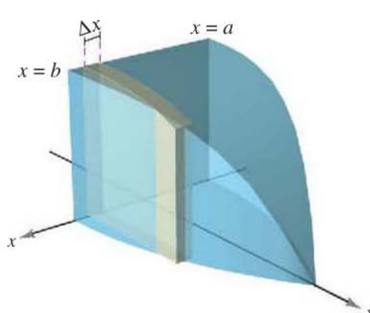
VOLUMES OF SOLIDS WITH KNOWN CROSS SECTIONS

1. For cross sections of area $A(x)$ taken perpendicular to the x-axis,

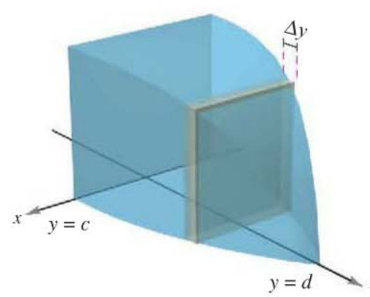
$$\text{Volume} = \int_a^b A(x)\,dx.$$ See Figure 7.24(a).

2. For cross sections of area $A(y)$ taken perpendicular to the y-axis,

$$\text{Volume} = \int_c^d A(y)\,dy.$$ See Figure 7.24(b).

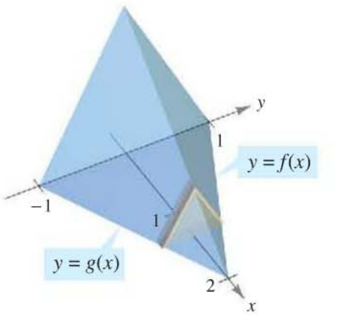


(a) Cross sections perpendicular to x-axis

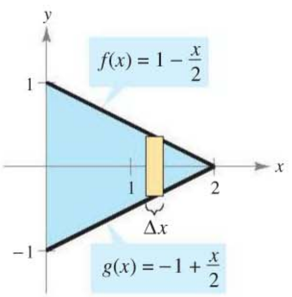


(b) Cross sections perpendicular to y-axis

Figure 7.24



Cross sections are equilateral triangles.



Triangular base in *xy*-plane

Figure 7.25

EXAMPLE 6 Triangular Cross Sections

Find the volume of the solid shown in Figure 7.25. The base of the solid is the region bounded by the lines

$$f(x) = 1 - \frac{x}{2}, \qquad g(x) = -1 + \frac{x}{2}, \qquad \text{and} \qquad x = 0.$$

The cross sections perpendicular to the x-axis are equilateral triangles.

Solution The base and area of each triangular cross section are as follows.

$$\text{Base} = \left(1 - \frac{x}{2}\right) - \left(-1 + \frac{x}{2}\right) = 2 - x \qquad \text{Length of base}$$

$$\text{Area} = \frac{\sqrt{3}}{4}(\text{base})^2 \qquad \text{Area of equilateral triangle}$$

$$A(x) = \frac{\sqrt{3}}{4}(2 - x)^2 \qquad \text{Area of cross section}$$

Because x ranges from 0 to 2, the volume of the solid is

$$V = \int_a^b A(x)\,dx = \int_0^2 \frac{\sqrt{3}}{4}(2 - x)^2\,dx$$

$$= -\frac{\sqrt{3}}{4}\left[\frac{(2 - x)^3}{3}\right]_0^2 = \frac{2\sqrt{3}}{3}.$$

EXAMPLE 7 An Application to Geometry

Prove that the volume of a pyramid with a square base is $V = \frac{1}{3}hB$, where h is the height of the pyramid and B is the area of the base.

Solution As shown in Figure 7.26, you can intersect the pyramid with a plane parallel to the base at height y to form a square cross section whose sides are of length b'. Using similar triangles, you can show that

$$\frac{b'}{b} = \frac{h - y}{h} \qquad \text{or} \qquad b' = \frac{b}{h}(h - y)$$

where b is the length of the sides of the base of the pyramid. So,

$$A(y) = (b')^2 = \frac{b^2}{h^2}(h - y)^2.$$

Integrating between 0 and h produces

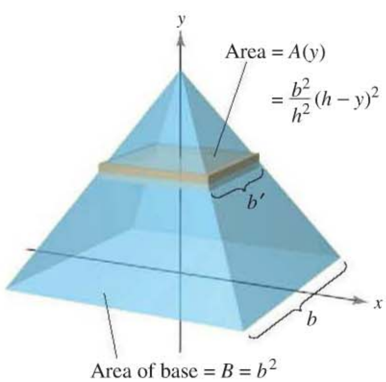
$$V = \int_0^h A(y)\,dy = \int_0^h \frac{b^2}{h^2}(h - y)^2\,dy$$

$$= \frac{b^2}{h^2}\int_0^h (h - y)^2\,dy$$

$$= -\left(\frac{b^2}{h^2}\right)\left[\frac{(h - y)^3}{3}\right]_0^h$$

$$= \frac{b^2}{h^2}\left(\frac{h^3}{3}\right)$$

$$= \frac{1}{3}hB. \qquad B = b^2$$



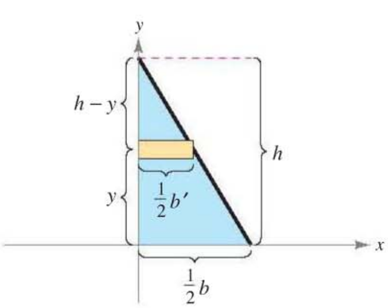


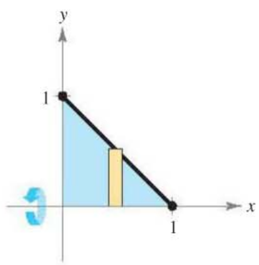
Figure 7.26

7.2 Exercises

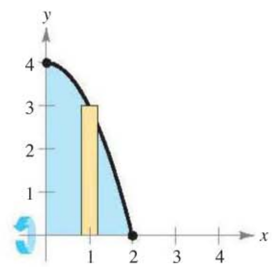
See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1–6, set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the x-axis.

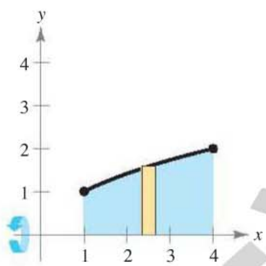
1. $y = -x + 1$



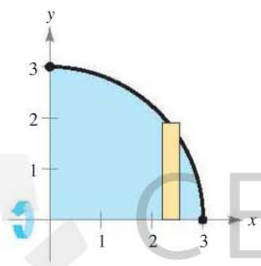
2. $y = 4 - x^2$



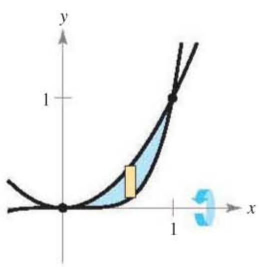
3. $y = \sqrt{x}$



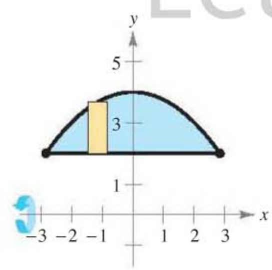
4. $y = \sqrt{9 - x^2}$



5. $y = x^2,\ y = x^5$

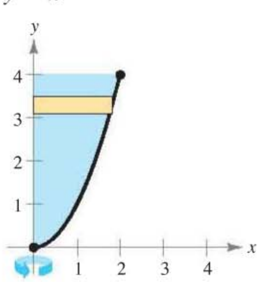


6. $y = 2,\quad y = 4 - \dfrac{x^2}{4}$

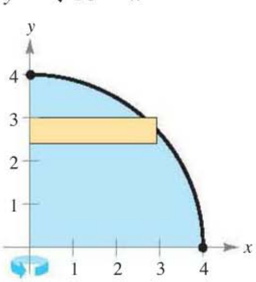


In Exercises 7–10, set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the y-axis.

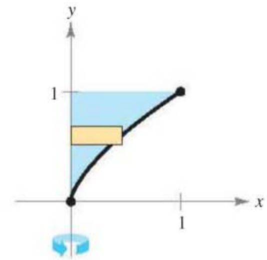
7. $y = x^2$



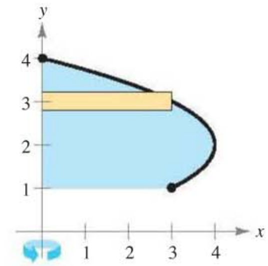
8. $y = \sqrt{16 - x^2}$



9. $y = x^{2/3}$



10. $x = -y^2 + 4y$



In Exercises 11–14, find the volumes of the solids generated by revolving the regions bounded by the graphs of the equations about the given lines.

11. $y = \sqrt{x},\quad y = 0,\quad x = 3$
    (a) the x-axis (b) the y-axis
    (c) the line $x = 3$ (d) the line $x = 6$

12. $y = 2x^2,\quad y = 0,\quad x = 2$
    (a) the y-axis (b) the x-axis
    (c) the line $y = 8$ (d) the line $x = 2$

13. $y = x^2,\quad y = 4x - x^2$
    (a) the x-axis (b) the line $y = 6$

14. $y = 6 - 2x - x^2,\quad y = x + 6$
    (a) the x-axis (b) the line $y = 3$

In Exercises 15–18, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line $y = 4$.

15. $y = x,\quad y = 3,\quad x = 0$

16. $y = \frac{1}{2}x^3,\quad y = 4,\quad x = 0$

17. $y = \dfrac{3}{1 + x},\quad y = 0,\quad x = 0,\quad x = 3$

18. $y = \sec x,\quad y = 0,\quad 0 \le x \le \dfrac{\pi}{3}$

In Exercises 19–22, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line $x = 5$.

19. $y = x,\quad y = 0,\quad y = 4,\quad x = 5$

20. $y = 5 - x,\quad y = 0,\quad y = 4,\quad x = 0$

21. $x = y^2,\quad x = 4$

22. $xy = 5,\quad y = 2,\quad y = 5,\quad x = 5$

In Exercises 23–30, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis.

23. $y = \dfrac{1}{\sqrt{x + 1}},\quad y = 0,\quad x = 0,\quad x = 4$

24. $y = x\sqrt{9 - x^2},\quad y = 0$

25. $y = \dfrac{1}{x}, \quad y = 0, \quad x = 1, \quad x = 3$

26. $y = \dfrac{2}{x+1}, \quad y = 0, \quad x = 0, \quad x = 6$

27. $y = e^{-x}, \quad y = 0, \quad x = 0, \quad x = 1$

28. $y = e^{x/2}, \quad y = 0, \quad x = 0, \quad x = 4$

29. $y = x^2 + 1, \quad y = -x^2 + 2x + 5, \quad x = 0, \quad x = 3$

30. $y = \sqrt{x}, \quad y = -\frac{1}{2}x + 4, \quad x = 0, \quad x = 8$

In Exercises 31 and 32, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the *y*-axis.

31. $y = 3(2 - x), \quad y = 0, \quad x = 0$

32. $y = 9 - x^2, \quad y = 0, \quad x = 2, \quad x = 3$

In Exercises 33–36, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the *x*-axis. Verify your results using the integration capabilities of a graphing utility.

33. $y = \sin x, \quad y = 0, \quad x = 0, \quad x = \pi$

34. $y = \cos 2x, \quad y = 0, \quad x = 0, \quad x = \dfrac{\pi}{4}$

35. $y = e^{x-1}, \quad y = 0, \quad x = 1, \quad x = 2$

36. $y = e^{x/2} + e^{-x/2}, \quad y = 0, \quad x = -1, \quad x = 2$

In Exercises 37–40, use the integration capabilities of a graphing utility to approximate the volume of the solid generated by revolving the region bounded by the graphs of the equations about the *x*-axis.

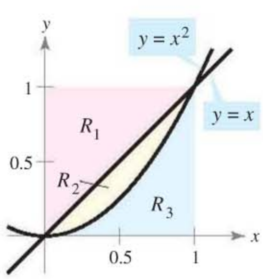
37. $y = e^{-x^2}, \quad y = 0, \quad x = 0, \quad x = 2$

38. $y = \ln x, \quad y = 0, \quad x = 1, \quad x = 3$

39. $y = 2 \arctan(0.2x), \quad y = 0, \quad x = 0, \quad x = 5$

40. $y = \sqrt{2x}, \quad y = x^2$

In Exercises 41–48, find the volume generated by rotating the given region about the specified line.



41. R_1 about $x = 0$

42. R_1 about $x = 1$

43. R_2 about $y = 0$

44. R_2 about $y = 1$

45. R_3 about $x = 0$

46. R_3 about $x = 1$

47. R_2 about $x = 0$

48. R_2 about $x = 1$

Think About It **In Exercises 49 and 50, determine which value best approximates the volume of the solid generated by revolving the region bounded by the graphs of the equations about the *x*-axis. (Make your selection on the basis of a sketch of the solid and *not* by performing any calculations.)**

49. $y = e^{-x^2/2}, \quad y = 0, \quad x = 0, \quad x = 2$

(a) 3 (b) −5 (c) 10 (d) 7 (e) 20

50. $y = \arctan x, \quad y = 0, \quad x = 0, \quad x = 1$

(a) 10 (b) $\frac{3}{4}$ (c) 5 (d) −6 (e) 15

WRITING ABOUT CONCEPTS

In Exercises 51 and 52, the integral represents the volume of a solid. Describe the solid.

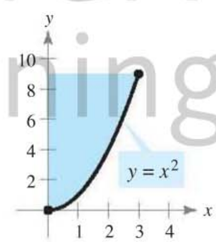
51. $\pi \displaystyle\int_0^{\pi/2} \sin^2 x \, dx$

52. $\pi \displaystyle\int_2^4 y^4 \, dy$

53. A region bounded by the parabola $y = 4x - x^2$ and the x-axis is revolved about the x-axis. A second region bounded by the parabola $y = 4 - x^2$ and the x-axis is revolved about the x-axis. Without integrating, how do the volumes of the two solids compare? Explain.

54. The region in the figure is revolved about the indicated axes and line. Order the volumes of the resulting solids from least to greatest. Explain your reasoning.

(a) x-axis (b) y-axis (c) $x = 3$



55. Discuss the validity of the following statements.

(a) For a solid formed by rotating the region under a graph about the x-axis, the cross sections perpendicular to the x-axis are circular disks.

(b) For a solid formed by rotating the region between two graphs about the x-axis, the cross sections perpendicular to the x-axis are circular disks.

CAPSTONE

56. Identify the integral that represents the volume of the solid obtained by rotating the area between $y = f(x)$ and $y = g(x)$, $a \le x \le b$, about the x-axis. [Assume $f(x) \ge g(x) \ge 0$.]

(a) $\pi \displaystyle\int_a^b [f(x) - g(x)]^2 \, dx$ (b) $\pi \displaystyle\int_a^b \left([f(x)]^2 - [g(x)]^2\right) dx$

57. If the portion of the line $y = \frac{1}{2}x$ lying in the first quadrant is revolved about the x-axis, a cone is generated. Find the volume of the cone extending from $x = 0$ to $x = 6$.

58. Use the disk method to verify that the volume of a right circular cone is $\frac{1}{3}\pi r^2 h$, where r is the radius of the base and h is the height.

59. Use the disk method to verify that the volume of a sphere is $\frac{4}{3}\pi r^3$.

60. A sphere of radius r is cut by a plane h ($h < r$) units above the equator. Find the volume of the solid (spherical segment) above the plane.

61. A cone of height H with a base of radius r is cut by a plane parallel to and h units above the base. Find the volume of the solid (frustum of a cone) below the plane.

62. The region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$ is revolved about the x-axis.

(a) Find the value of x in the interval $[0, 4]$ that divides the solid into two parts of equal volume.

(b) Find the values of x in the interval $[0, 4]$ that divide the solid into three parts of equal volume.

63. ***Volume of a Fuel Tank*** A tank on the wing of a jet aircraft is formed by revolving the region bounded by the graph of $y = \frac{1}{8}x^2\sqrt{2 - x}$ and the x-axis $(0 \le x \le 2)$ about the x-axis, where x and y are measured in meters. Use a graphing utility to graph the function and find the volume of the tank.

64. ***Volume of a Lab Glass*** A glass container can be modeled by revolving the graph of

$$y = \begin{cases} \sqrt{0.1x^3 - 2.2x^2 + 10.9x + 22.2}, & 0 \le x \le 11.5 \\ 2.95, & 11.5 < x \le 15 \end{cases}$$

about the x-axis, where x and y are measured in centimeters. Use a graphing utility to graph the function and find the volume of the container.

65. Find the volumes of the solids (see figures) generated if the upper half of the ellipse $9x^2 + 25y^2 = 225$ is revolved about (a) the x-axis to form a prolate spheroid (shaped like a football), and (b) the y-axis to form an oblate spheroid (shaped like half of a candy).

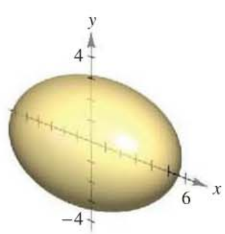


Figure for 65(a)

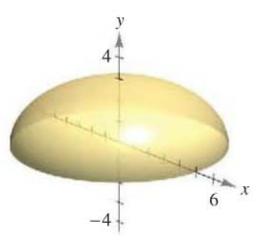


Figure for 65(b)

66. ***Water Depth in a Tank*** A tank on a water tower is a sphere of radius 50 feet. Determine the depths of the water when the tank is filled to one-fourth and three-fourths of its total capacity. (*Note:* Use the *zero* or *root* feature of a graphing utility after evaluating the definite integral.)

67. ***Minimum Volume*** The arc of $y = 4 - (x^2/4)$ on the interval $[0, 4]$ is revolved about the line $y = b$ (see figure).

(a) Find the volume of the resulting solid as a function of b.

(b) Use a graphing utility to graph the function in part (a), and use the graph to approximate the value of b that minimizes the volume of the solid.

(c) Use calculus to find the value of b that minimizes the volume of the solid, and compare the result with the answer to part (b).

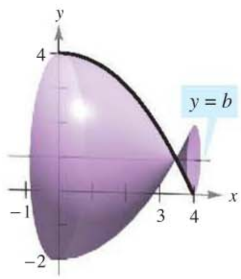


Figure for 67

Figure for 68

68. ***Modeling Data*** A draftsman is asked to determine the amount of material required to produce a machine part (see figure). The diameters d of the part at equally spaced points x are listed in the table. The measurements are listed in centimeters.

x	0	1	2	3	4	5
d	4.2	3.8	4.2	4.7	5.2	5.7

x	6	7	8	9	10
d	5.8	5.4	4.9	4.4	4.6

(a) Use these data with Simpson's Rule to approximate the volume of the part.

(b) Use the regression capabilities of a graphing utility to find a fourth-degree polynomial through the points representing the radius of the solid. Plot the data and graph the model.

(c) Use a graphing utility to approximate the definite integral yielding the volume of the part. Compare the result with the answer to part (a).

69. ***Think About It*** Match each integral with the solid whose volume it represents, and give the dimensions of each solid.

(a) Right circular cylinder (b) Ellipsoid

(c) Sphere (d) Right circular cone (e) Torus

(i) $\pi\displaystyle\int_0^h \left(\frac{rx}{h}\right)^2 dx$ (ii) $\pi\displaystyle\int_0^h r^2\,dx$

(iii) $\pi\displaystyle\int_{-r}^{r} \left(\sqrt{r^2 - x^2}\right)^2 dx$ (iv) $\pi\displaystyle\int_{-b}^{b} \left(a\sqrt{1 - \frac{x^2}{b^2}}\right)^2 dx$

(v) $\pi\displaystyle\int_{-r}^{r} \left[\left(R + \sqrt{r^2 - x^2}\right)^2 - \left(R - \sqrt{r^2 - x^2}\right)^2\right] dx$